

Exercise 21

Find the gradient vector field of f .

$$f(x, y) = y \sin(xy)$$

Solution

Calculate the gradient and call it \mathbf{F} .

$$\begin{aligned} \mathbf{F} &= \nabla f \\ &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle f \\ &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \\ &= \left\langle \frac{\partial}{\partial x}[y \sin(xy)], \frac{\partial}{\partial y}[y \sin(xy)] \right\rangle \\ &= \left\langle \left[\frac{\partial}{\partial x}(y) \right] \sin(xy) + y \frac{\partial}{\partial x}[\sin(xy)], \left[\frac{\partial}{\partial y}(y) \right] \sin(xy) + y \frac{\partial}{\partial y}[\sin(xy)] \right\rangle \\ &= \left\langle (0) \sin(xy) + y \left[\cos(xy) \cdot \frac{\partial}{\partial x}(xy) \right], (1) \sin(xy) + y \left[\cos(xy) \cdot \frac{\partial}{\partial y}(xy) \right] \right\rangle \\ &= \langle y [\cos(xy) \cdot (y)], \sin(xy) + y [\cos(xy) \cdot (x)] \rangle \\ &= \langle y^2 \cos(xy), \sin(xy) + xy \cos(xy) \rangle \end{aligned}$$

The vector field of this gradient is superimposed on a contour plot of $f(x, y)$. Notice that the vectors are perpendicular to each of the contours, pointing in the direction of greatest increase.

